für die Jahre 1875–1895." This later part has not been included in the new edition.

In regard to the scope of the work, Sudhoff, in the preface to the second edition, says: "Dass ich persönlich unter 'Geschichte der Medizin' etwas mehr verstehe als eine medizinische Literaturgeschichte: eine kulturgeschichtliche Erfassung der heilenden Kunst und Wissenschaft im Gesamtleben der Zeiten, dürfte bekannt sein, kommt aber hier nicht in Frage, wo es sich um eine 'Einführung,' um ein Lehrbuch der Medizingeschichte handelt."

Pagel, likewise, has a broad idea of the importance of the history of medicine, for he says: "Die ganze moderne Medizin baut sich auf dem Gedanken der Entwicklung auf."

As the title indicates, the volume was based originally on a series of lectures, more or less popular in nature, but all of them readable. The lectures are a little more thorough in their content than those of Ernst Schwalbe[1] and the additions made by Karl Sudhoff raises it out of the ranks of a volume of lectures and forms the greater part of my excuse for reviewing the work in this place.

The work proceeds along well-defined and usual lines, taking up serially the development of medicine in the various countries. There is nothing new or startling in the method of their presentation, but the facts are essentially all there and the addenda and references by the editor make the book a most useful one for the beginning student.

The first lecture deals with the beginnings of the healing art and discusses the nature of medical work in ancient and modern China and Japan and among the Aztecs of Mexico. The second lecture discusses medical history among the peoples of ancient India, Babylonia, Egypt, Palestine and the other countries of Asia Minor. The succeeding four lectures are devoted to the medical lore of the Greeks, with one entire chapter given to Galen.

The lectures from this point take up the development of modern medicine, and the later lectures are given a more biographical cast as various eminent men exerted an influence over various phases of medical work. Interpolated throughout these pages there is given by Sudhoff, in a way to be found nowhere else, the sources of information, recent developments of each special topic and recent literature, but not in such abundance as to be tiresome to the general reader. So that in addition to being a volume of very readable lectures it may also be used as a work of reference of no small importance, though of course not attempting to rank with the Handbücher of Pagel and Häser. It will appeal to the general reader as being free from a number of technicalities and will be found to be one of the best one-volume presentations of medical history of recent years.

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[1] "Vorlesungen ueber Geschichte der Medizin," Jena, 1909.

THE ZERO AND PRINCIPLE OF LOCAL VALUE USED BY THE MAYA OF CENTRAL AMERICA

Historians of mathematics refer to the vigesimal system of the Maya[1] of Central America and southern Mexico, but, to my knowledge, no historian conveys the information that the Maya, in the writing of numbers, employed symbols for zero and the principle of local value. Added interest attaches to this matter from the fact that the Maya appear to have done this earlier than any one else. My attention is called to this achievement of the Maya by a recent book issued by the Government Printing Office in Washington, entitled *An Introduction to the Study of the Maya Hieroglyphs,* by Sylvanus Griswold Morley, 1915. This publication constitutes Bulletin 57 of the Bureau of American Ethnology. Nearly all the information contained in this article is drawn from that source.

The age of the Maya inscriptions and codices is a matter of vital interest and, as yet, of considerable doubt. It is known that

[1] See, for instance, M. Cantor, "Geschichte der Mathematik," Vol. I., 3d ed., 1907, p. 9.

all dated monuments had their origin within 400 years of each other. The Maya had an accurate system of chronology, but the difficulty lies in establishing a correlation between their chronology and our own. Authorities differ on this point. Take one of the monuments, called Stela 9, in the ancient town of Copan in Honduras. Mr. Morley summarizes the various conclusions regarding the date for Stela 9 thus:[2]

> Professor Seler's date of 1255 B.C. for this is by far the oldest; Mr. Bowditch's date, A.D. 34, comes next. My own correlation assigns a date to this monument somewhere between the years 284 to 304 A.D., which an assumption made by both Mr. Bowditch and Professor Seler in their correlations would narrow to A.D. 294. Finally, the passage from The Book of Chilan Balam of Mani, as I have amended it, gives the date of this monument as A.D. 282.

The Ethnologist-in-Charge, F. W. Hodge, in his "Letter of Transmittal" of Morley's book expresses himself thus:

> The earliest inscriptions now extant probably date from about the beginning of the Christian era, but such is the complexity of the glyphs and subject-matter even at this early period, that in order to estimate the age of the system it is necessary to postulate a far greater antiquity for its origin.

For purposes of comparison, let us recall the dates of the number systems of the Babylonians and Hindus. The early Babylonians possessed the principle of local value, but, so far as now known, did not possess a zero. About two centuries B.C. they did have a zero-symbol, which was "not used in calculation, nor does it always occur when units of any order are lacking."[3] They did not employ it systematically in writing numbers and not at all in performing computations. The Hindus certainly did not use their zero-symbol systematically in their decimal number-system before probably the sixth century A.D.; the earliest undoubted occurrence of our zero in India is A.D. 876. Mr. G. R. Kaye[4] mentions A.D. 595 and A.D. 662 as dates when, as claimed by some, Indian figures were known; "on the other hand it is held that there is no sound evidence of the employment in India of a place-value system earlier than about the ninth century."

In view of this, special interest attaches to the occurrence of zero-symbols and the principle of local value among the inhabitants of the flat lands of Central America, at a period as early as the beginning of the Christian era, if not much earlier. It would seem that in this invention, the Maya in Central America possessed priority over the Asiatic peoples by a margin of five or six centuries.

The Maya number system is remarkable for the extent of its early development. Records of Maya calendars and chronology are numerous and have been successfully deciphered. In fact, "it must be admitted that very little progress has been made in deciphering the Maya glyphs except those relating to the calendar and chronology; that is, the signs for the various time periods (days and months), the numerals, and a few name-glyphs; however, as these known signs comprise possibly two fifths of all the glyphs, it is clear that the general tenor of the Maya inscriptions is no longer concealed from us."[5] As far as known, the Maya used their numeral systems only in the counting of time, as it arose in their calendar, ritual and astronomy. Many numbers that are found in inscriptions and codices occur in connection with signs, the meanings of which have not yet been ascertained. Hence, after the meanings of more glyphs are deciphered, it may be found that the numeral system had much wider application than is evident at present.

Of the several Maya numeral notations we briefly describe the one which is of greatest interest to mathematicians on account of its embodying the principle of local value and the use of symbols for zero. It is found in Maya codices, but not in their inscriptions. The

[2] S. G. Morley, "The Correlation of Maya and Christian Chronology," *American Journal of Archæology*, Vol. 14, 1910, p. 204.

[3] D. E. Smith and L. C. Karpinski, "Hindu-Arabic Numerals," 1911, p. 51.

[4] G. R. Kaye, "Indian Mathematics," Calcutta and Simla, 1915, p. 31.

[5] S. G. Morley, *op. cit.*, p. iv.

ratio of increase of successive units in this and the other fully developed Maya systems was not 10, as in the Hindu-Arabic system; it was 20 in all positions except the third. That is, 20 units of the lowest order (*kins,* or days) make one unit of the next higher order (*uinal,* or 20 days), 18 uinals make one unit of the third order (*tun,* or 360 days), 20 tuns make one unit of the fourth order (*katun,* or 7,200 days), 20 katuns made one unit of the fifth order (*cycle,* or 144,000 days), and finally, 20 cycles make one *great cycle* of 2,-880,000 days. It has been contended by some archeologists that in Maya inscriptions, not 20, but 13, *cycles* constitute a *great cycle,* but in the Maya codices all archeologists agree that the only break in the vigesimal system lies in the relation that 18 uinals equal 1 tun. Proceeding now to the notation, as found in the codices, we find symbols 1 to 19, both inclusive, expressed by bars and dots. Each bar stands for five units, each dot for 1 unit. For instance,

1 2 4 5 7 11 19

The values of the bars and dots are *added* in each case. The zero, which plays a leading part in the notations found on inscriptions as well as those on codices, is represented in the codices by a symbol that looks roughly like a half-closed eye. This zero and the symbols for 1 — 19 in the Maya vigesimal notation correspond to the symbols 0, 1, 2, . . . 9 in our decimal notation. In writing 20, in the Maya codices, the principle of local value enters for the first time. It is expressed by a dot placed over the symbol for zero. The numerals are written, not horizontally, but vertically, the unit of lowest order or value being assigned the lowest position. Accordingly, 37 was expressed by the symbols for 17 (three bars and two dots) in the kin place and one dot, representing 20, placed above the 17, in the uinal place. The number 300 is expressed by three bars drawn above the symbol for zero ($3 \times 5 \times 20 = 300$). The largest number which can be written by the use of only two places or positions is $17 \times 20 + 19 = 359$. To write 360, the Maya drew two zeros, one above the other, with one dot higher up, in third place. Using three places to represent *kins, uinals* and *tuns,* they could write any number not larger than 7,199. Proceeding in this way the Maya wrote numbers in very compact form. The highest number found in the codices is 12,489,781. It occurs on page 61 of what is known as the "Dresden Codex," a fiber-paper booklet that was reproduced facsimile by Professor E. Förstemann in 1880 and 1892. The symbols representing this number occupy six different places, one above the other. Proceeding from bottom up, the symbols in the six places are, respectively, one dot, three bars, two bars and three dots, two bars and four dots, one bar and one dot, four dots. Thus the numerals in the six places are, respectively, 1, 15, 13, 14, 6, 4. Applying to these the principle of local value, they represent altogether: $1 + 15 \times 20 + 13 \times 18 \times 20 + 14 \times 20 \times 18 \times 20 + 6 \times 20 \times 20 \times 18 \times 20 + 4 \times 20 \times 20 \times 20 \times 18 \times 20 = 12{,}489{,}781$. From these illustrations it is seen that the Maya used the zero and the principle of local value consistently in the writing of numbers reaching into the millions.

The second numeral notation that was fully developed and employed by the Maya is found in their inscriptions. It employs the zero, but not the principle of local value. Special glyphs are employed to designate the different units. It is as if we were to write 1203 as "1 thousand, 2 hundred, 0 tens, 3 ones." We omit a detailed description of the system. The ratios of successive orders of units are the same as in the preceding, with the exception, perhaps, of the unit of the sixth order. In this second notation, that unit may rest upon the ratio 13, instead of 20, as we stated above.

The numerals in the Maya codices appear to the present writer to disclose traces of an imperfect quinary system, as seen in the use of the bar to represent 5. Similarly it seems to the present writer that there is a trace of an imperfect decimal system in Maya numerals found in inscriptions, where 16 — 19 are represented by two symbols, one symbol for 10 and the other for 6, 7, 8, 9, respectively.

We shall not attempt to describe the Maya chronology. It is a complicated and highly developed system. The larger part of Morley's book is devoted to the description of it. His exposition is admirably clear. No specimens of Maya computation are extant. Maya records contain only the results of computation. It is evident that considerable reckoning is involved in Maya chronology. The Maya had a sacred year of 260 days, an official year of 360 days and a solar year of 365 + days. The fact that $360 = 18 \times 20$ seems to account for the break in the vigesimal system, making 18 (rather than 20) uinals equal to 1 tun. Apparently, the Maya found the lowest common multiple of 260 and 365, or 18,980. In their calendar 18,980 days consituted the "Calendar Round," a period of 52 years which is "the most important period of Maya chronology." Using this period, the Maya developed an elaborate system of counting time, "wherein any date of the Calendar Round could be fixed with absolute certainty within a period of 374,400 years."

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SPECIAL ARTICLES

THE FOCUS OF THE AURORAL STREAMERS ON AUGUST 26, 1916

IN a recent number of SCIENCE[1] the remarkable auroral display of August 26 was described by Professor C. C. Nutting, as observed by him at Lake Douglas in northern Michigan. The phenomenon was reported to have been of unusual intensity and beauty. The appearance of streamers in the southern sky was particularly noted, as well as the fact that the auroral glow prevailed around the entire vault of the heavens, causing the earth to be illuminated without shadows.

This aurora was widespread because it was also seen in northern New York, in New Hampshire, in Nova Scotia, and over the Gulf of St. Lawrence. According to a letter in the current issue of SCIENCE,[2] it was observed as far south as Martha's Vineyard, Mass. In each case the characteristics so well described by Professor Nutting were observed.

It has been reported as far west as Washta, Ia., by F. S. Carrington.[3] In this case the streamers in the northeast passed to the south of the zenith, and the glow in the southern horizon reached to about 30°.

The aurora evidently extended eastward to the British Isles, because a bright display was reported by Mr. W. F. Denning at Bristol, England, from 2 to 4 A.M., August 27. The streamers were observed to an altitude of 70° in the northern sky, and moved rapidly from west to east.[4]

It was seen at Eskdalemuir, Dumfriesshire, from 9 P.M., August 26, to past midnight, accompanied by considerable disturbance of the magnets at the Kew Observatory. The magnetic storm commenced suddenly at 7:45 P.M., August 26. It was observed at Seskin, Waterford, in Ireland, from 10:05 to 10:40 P.M., August 26, the streamers in the northern sky stretching to within 20° or 30° of the zenith.[5]

The aurora was seen on the north shore of Prince Edward Island by the writer, who noted some of its interesting features; among which was the location of the apparent focus of the auroral streamers with respect to some readily identified stars. To this particular attention was paid.

GENERAL FEATURES OF THE AURORA

The writer was on a wide stretch of water and observed the beginning of the aurora, which occurred at 8:15 P.M. Atlantic time, the sky being perfectly clear. The glow at first showed dimly in the southern sky, but rapidly increased in intensity until the entire southern portion of the vault of the heavens was pierced by pale greenish lance-like streamers. Those overhead terminated in a well defined focus, southeast of the zenith, as shown in Fig. 1.

For some minutes there was no evidence whatever of an aurora to the north. Later, streamers rose in that section, and soon the

[1] N. S., Vol. XLIV., October 6, 1916.
[2] N. S., Vol. XLIV., October 20, 1916.
[3] *The Guide to Nature*, November, 1916, p. 191.
[4] *Nature*, Vol. 97, 2444, August 31, 1916.
[5] *Nature*, Vol. 98, 2447, September 21, 1916.